PROPAGATION OF ELECTROMAGNETIC DISTURBANCES ABOVE A PLANE SURFACE

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Under certain assumptions, it is shown that the propagation problem for an electromagnetic disturbance becomes self-similar, and the self-similarity parameters are determined. A basis is given for the absence of reflection, and it is shown that it is equivalent to the boundary conditions of M. A. Leontovich. Solutions of the propagation problem are obtained for the various components of a pulsed signal field from a dipole of arbitrary orientation, and their properties studied.

Solution of the problem involving propagation of an electromagnetic pulse along a plane-impedence surface is of interest in the interpretation of observational results for electromagnetic fields varying over times of the order of 10 μsec. There is great interest in a determination of the impulse function for a propagation pattern which describes the distortion during the propagation of a δ-shaped pulse. The impulse function has been studied [1-3] for the particular case of ground location of source and point of observation. The source is assumed to be a point vertical dipole and the pattern is a plane surface with conductivity $\sigma = 1/4\gamma\pi^{-1}$ and dielectric constant ε.

The existence of a reciprocity theorem [4] means that for a complete solution of the problem involving the distortion of the field of an arbitrarily oriented dipole it is necessary to determine four functions: K_{vv}, K_{hv}, K_{hh}, and M, which are respectively the vertical component of a dipole polarized along the normal to the surface, the vertical and horizontal components of a dipole polarized in the plane of polarization parallel to the ground surface and of a dipole polarized along the normal to the plane of propagation not confined to the case of surface propagation.

Only the properties of the functions K_{vv}, K_{hv}, and K_{hh} are considered in the following because the function M at practically all heights reduces to the corresponding Fresnelcoefficient in the time representation. The attenuation functions of the fields from point monochromatic dipoles have been introduced as dimensionless quantities [4, 5]. Therefore the impulse functions have the dimension of inverse time. As a consequence, if the solution of the propagation problem for a δ-pulse signal possesses self-similarity, the impulse function K(t) will not be self-similar and the transfer function

$$P(t) = \int_{-\infty}^{t} K(t')\,dt'$$

1. Self-Similarity of Solution. Characteristic Time of Impulse Function. The signal at the point of observation consists of a direct and reflected part. The properties of the direct signal are obvious: its propagation function is $\delta(t - R/c)$, where R is the distance from source to point of observation and c is the velocity of light in vacuum. In the case of a pattern symmetric with respect to the points of radiation and detection, the signal depends only on the sum of the heights of the source and detector and not on each of them individually. In the following, therefore, the source is assumed to be located on the surface at the origin of a cylindrical coordinate system ρ, φ, and z, with the z axis normal to the surface. In this case, the problem contains one less dimensional parameter. The transfer function will be a function of two

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variables if the Leontovich condition [6] is applicable and displacement currents can be neglected. To be specific, P(t) in this situation satisfies the equations

$$\frac{\partial^2 P}{\partial t^2} - c^2 \frac{\partial^2 P}{\partial z^2} - \frac{1}{\rho} \frac{\partial}{\partial \rho}\left(\rho \frac{\partial P}{\partial \rho}\right) = 0, \quad \left(\frac{\partial P}{\partial z}\right)_{z=0} = \frac{1}{c} \int_0^t \left(\frac{\partial^2 P}{\partial t'^2}\right)_{z=0} \frac{dt'}{[\pi\gamma(t-t')]^{1/2}} \tag{1.1}$$

For small heights, $R = \rho + z^2/2\rho$. We shift the variables in (1.1) to

$$\tau = t - \frac{\rho}{c} - \frac{z^2}{2\rho c}, \quad \eta = t + \frac{\rho}{c} + \frac{z^2}{2\rho c} \tag{1.2}$$

Equation (1.1) then takes the form

$$\frac{\partial^2 P}{\partial z^2} - \frac{4}{c^2} \frac{\partial^2 P}{\partial \tau \partial \eta} - \frac{2z}{c\rho} \frac{\partial^2 P}{\partial z \partial \tau} - \frac{2}{c\rho} \frac{\partial P}{\partial \tau} = 0$$
$$\left(\frac{\partial P}{\partial z}\right)_{z=0} = \frac{1}{c} \int_0^\tau \left(\frac{\partial^2 P}{\partial \tau'^2}\right)_{z=0} \frac{d\tau'}{[\pi\gamma(\tau-\tau')]^{1/2}} \tag{1.3}$$

We introduce the dimensionless variables

$$\tau_1 = a\tau, \; \eta_1 = b\eta, \; z_1 = dz, \; b / da^{1/2} = 1, \; a^2 / b = 1$$

Equation (1.3) is invariant with respect to such a substitution. Under those circumstances, one of the quantities a, b, or d can be chosen arbitrarily. Setting $a = t^{-1}$, we obtain

$$P(\rho, z, t) = P(1, \rho t^{-2}, z t^{3/2})$$

i.e., the transfer function depends either on the quantities ρt^{-2} and $z t^{3/2}$ or on arbitrary combinations of them. In the following, we use the combinations

$$A = t\left(\frac{c\gamma}{2\rho}\right)^{1/2}, \quad B = \frac{z(\gamma t)^{1/2}}{2\rho} \tag{1.4}$$

It is then obvious that the characteristic times for variation of the impulse function will be

$$t_1 = \frac{4\rho^2}{z^2\gamma}, \quad t_2 = \frac{1}{c}\sqrt{2}\rho\alpha^2, \quad \alpha = \left(\frac{c}{\gamma\rho}\right)^{1/4}$$

i.e., the analysis made above is valid if $\alpha^2 \ll 1$ and $z/\rho \ll 1$. The condition $\alpha^2 \ll 1$ is satisfied at distance greater than 300 m from the source for the usual terrestrial values of γ. As shown below, the condition $z/\rho \ll 1$ is of no practical importance because at heights for which it is satisfied the solution practically agrees with the propagation of a plane-wave pulse, which is analyzed by means of the Fresnel coefficient [7].

In the variables A and B, Eq. (1.3) takes the form

$$\frac{\partial F}{\partial A} + A \frac{\partial^2 F}{\partial A^2} + \frac{B}{2} \frac{\partial^2 F}{\partial A \partial B} + \frac{A}{2} \frac{\partial^2 F}{\partial B^2} = 0$$
$$\left(\frac{\partial F}{\partial B}\right)_{B=0} = \frac{1}{\sqrt{A}} \int_0^A \left(\frac{\partial^2 F}{\partial A'^2}\right)_{B=0} \frac{dA'}{[\pi(A-A')]^{1/2}}, \quad F(A, B) = \rho P(A, B) \; (1.6) \tag{1.5}$$

We clarify the meaning of the times t_1 and t_2. With the source and detector located at ground level, the parameter B = 0 and the transfer function is determined by the parameter A alone, which agrees with the results in [1], i.e., it is sufficient to analyze the case of surface propagation of the signal to clarify the meaning of A. On the other hand, when $\rho \to \infty$ so that $z/\rho = \text{const}$, $A \to 0$, $B = \text{const}$, i.e., the solution will be determined by the parameter B. In that case, the dipole field near the point of observation differs little from the field of a plane wave. Therefore the solution F(0, B) describes the distortion of a plane-wave pulse during reflection from the vacuum–soil interface.

The reflected signal is radiated by currents produced in the soil having a direction which is related both to the angle of incidence and the characteristic properties of the soil. Therefore it decreases until the time when the current direction becomes parallel to the re-radiation direction, i.e., the condition

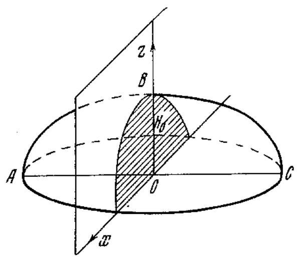

Fig. 1

$E_{z1}/E_\rho = \sin\psi$ is satisfied (the subscript 1 indicates the field in the soil and $\sin\psi = z/\rho$). However, $E_{z1} = \gamma^{-1}\partial E_z/\partial t$ results from the boundary condition $E_1 = E/\varepsilon$, and we correspondingly have $E_\rho \sim E_z/(\gamma t)^{1/2}$ from the Leontovich condition. It then follows that the reflected signal falls during a time determined by the condition $\sqrt{\gamma t}\sin\psi = 1$, which agrees with the condition B = 1.

To explain the meaning of A, we note that, for signal propagation along the surface, the only radiation managing to reach the point of observation at the time t is that which is propagated within an ellipsoid defined by the following condition: the distance from a point on the ellipsoid to the source plus the distance from the point on the ellipsoid to the point of observation is ct.

For short times, the minor axis of the ellipsoid (Fig. 1) $H_0 \sim (ct\rho)^{1/2}$. On the segments AB and BC, signal propagation occurs in purely wave fashion. Therefore the signal at the point of observation falls during a time t such that $\sqrt{\gamma t}\sin\psi \sim 1$, $\sin\psi \sim H_0(t)/\rho$, i.e., using the expression $H_0 \sim (ct\rho)^{1/2}$, during a time determined by the condition $t(c\gamma/2\rho)^{1/2} \sim 1$, which agrees with the condition A ~ 1.

Consequently, the transfer function will be self-similar in the variables A and B if the Leontovich condition and displacement currents can be neglected.

The resultant expression for the characteristic times, $t_1 = 1/\gamma\sin^2\psi$ and $t_2 = (2\rho/c\gamma)^{1/2}$, show that if a pattern must be analyzed on the basis of measurements, it is necessary to collect information with a resolution Δt no worse than t_2 for the determination of distance and with a resolution no worse than t_1 for the determination of height.

2. Modification of the Leontovich Boundary Condition. To analyze the effect of a surface on the signal, it is sufficient to determine the field at the surface itself. This problem is simpler than the determination of the fields in space because it contains one less parameter. There proceeds from space into the surface a diffraction energy flux with properties which are determined by the distribution of field energy in the neighborhood of the surface. At sufficiently large distances from the source, the surface itself dictates the structure of the leakage flux and the fields in the neighborhood of the surface. The mathematical expression of the situation are the Leontovich boundary conditions, which are equivalent to the following principle: that flux leaks into the surface which the surface is in a condition to absorb without a reflection. To accomplish this, it is necessary that the angle of incidence equal the Brewster angle [7] or the relation

$$E_\rho(\omega) = (\omega / i\gamma)^{1/2} E_z(\omega) \tag{2.1}$$

be satisfied.

However, this form of boundary condition is not always convenient because the quantity $\partial^2 E_z/\partial z^2$ appearing in the wave equation at the surface z = 0 is not defined for z = 0. From the fact the field depends only on $(t - \rho/c)/\rho^{1/2}$ for propagation along the surface, it follows that for $ct - \rho \ll \rho$ the equation div E = 0 takes the form

$$\frac{\partial E_z}{\partial z} = \frac{\partial E_\rho}{c\partial t} \tag{2.2}$$

Using Eq. (2.1), we then obtain

$$\left(\frac{\partial E_z}{\partial z}\right)_{z=0} = \frac{1}{c}\int_0^t \left(\frac{\partial^2 E_z}{\partial t'^2}\right)_{z=0} \frac{dt'}{[\pi\gamma(t-t')]^{1/2}} \tag{2.3}$$

Since Eq. (2.2) is true in some neighborhood of the surface, it can be differentiated with respect to z. In addition, because of the reciprocity theorem [4], condition (2.3) is also valid for $E_\rho(z, t)$, i.e., we obtain for $\partial^2 E_z/\partial z^2$

$$\left(\frac{\partial^2 E_z}{\partial z^2}\right)_{t,\rho} = \frac{1}{\gamma c^2}\frac{\partial^3 E_z}{\partial t^3} \tag{2.4}$$

This condition enables one to analyze the propagation of a pulse along the surface without solving the spatial problem. Note that since it is natural to study signal propagation as a function of time measured from the wave front, it is necessary to have a relation like Eq. (2.4) for the quantity $(\partial^2 E_z/\partial z^2)_{\tau,\rho}$.

Making the substitution of variables, we obtain

$$\left(\frac{\partial^2 E_z}{\partial z^2}\right)_{\tau,\rho} = \frac{1}{c^2\gamma}\frac{\partial^3 E_z}{\partial \tau^3} + \frac{1}{c\rho}\frac{\partial E_z}{\partial \tau} \tag{2.5}$$

3. Propagation Function For a Plane-Wave Pulse (at great height). At sufficiently great heights, the solution of the propagation problem for a δ-pulse signal is a function of the single parameter B. In this situation, the signal front can be considered to be plane. As seen in Fig. 2, $c\Delta t = \Delta z \sin\psi$ because we are transforming to the equivalent space-time point from the viewpoint of self-similarity.

The wave equation (1.5) is satisfied identically for A = 0 and the condition (1.6), after using the relation $c\,dt = \sin\psi\, dz$, leads to

$$F(B_1) = \int_0^{B_1} \frac{dF}{dB_1'}\frac{dB_1'}{[\pi(B_1 - B_1')]^{1/2}}, \qquad B_1 = B^2 \tag{3.1}$$

Solving (3.1) with the condition F(0) = 0, we obtain

$$F(B_1) = 2\left(1 - \frac{2e^{B_1}}{\sqrt{\pi}}\int_{\sqrt{B_1}}^{\infty} e^{-z^2}dz\right) \tag{3.2}$$

It is easy to show that the impulse function K(t) for a plane-wave signal corresponding to a given $F(B_1)$ agrees with the functions which is obtained from the Fresnel coefficient for a wave polarized in the plane of incidence by means of a Fourier transformation.

To explain the meaning of the propagation function, we consider the case of a plane-wave pulse incident along the normal to the surface. The boundary problem for soil currents j(x, t) takes the form

$$\begin{gathered}\frac{\gamma}{c^2}\cdot\frac{\partial j}{\partial t} = \frac{\partial^2 j}{\partial x^2}\\ j(x, 0) = 0, \qquad j(0, t) = \delta(t), \qquad x > 0\end{gathered} \tag{3.3}$$

(the x axis coincides with the normal from the vacuum into the medium).

The solution of such a problem takes the form

$$j(x, t) = \frac{x\sqrt{\gamma}}{2\sqrt{\pi}\,ct^{3/2}}\exp\left(-\frac{x^2\gamma}{4c^2 t}\right)$$

The reflected field at the point of observation at the time t − R/c is the field radiated by these currents

$$E(t) = \frac{1}{c}\frac{\partial}{\partial t}\frac{\sqrt{\gamma}}{2\sqrt{\pi}\,c}\int_0^{ct}\exp\left[-\frac{x^2\gamma}{4c^2(t - x/c)}\right]\frac{x\,dx}{(t - x/c)^{3/2}} \tag{3.4}$$

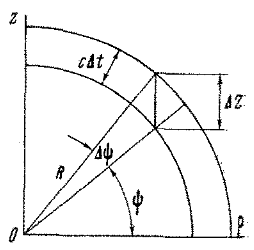

Fig. 2

This equation contains the parameter $x^2\gamma/4c^2t$. It is easily seen that this is the ratio of the depth of a point at which the current is being considered to the skin-layer depth $\delta = c(t/\gamma)^{1/2}$. In the case $\psi \neq \pi/2$, integration is carried out along the line of sight, and one should therefore equate $\delta/\sin\psi$ – the effective skin-layer depth – with x. It then follows that in the general case of angles $\psi \neq \pi/2$, the expression for the reflected field is obtained from (3.4) by the substitution $\gamma \to \gamma\sin^2\psi$, so that in the general case

$$K(t) = -\frac{\partial}{\partial t}\frac{2\sqrt{B_1}}{\sqrt{\pi}}\int_0^1 \exp\left[-\frac{x^2 B_1}{1 - x}\right]\frac{x\,dx}{(1 - x)^{3/2}} \tag{3.5}$$

This equation reduces to (3.2) by the substitution $z = x^2/(1-x)$.

We discuss the question of the applicability of the equation obtained. It is valid in cases where the slope angle of the wave front changes little over significant times, i.e., $\psi^{-1}\Delta\psi \ll 1$. Since $\psi^{-1}\Delta\psi \sim z^{-1}\Delta z$, then $z^{-1}\Delta z \sim \Delta t c\rho z^{-2}$, i.e., the signal can be considered as a plane wave up to the time $t_0 \sim z^2/c\rho$. It remains a plane wave for all times $t \ll 1/\gamma \sin^2\psi$, if $z \gg (\rho^3 c/\gamma)^{1/4}$.

The propagation function being discussed describes the behavior for reflection of the vertical component of the field of a vertical dipole. For a plane-wave signal, however, the z and ρ components of the field are related through the expression $E_\rho = -E_z \mathrm{tg}\psi$; $\mathrm{tg}\psi \approx \sin\psi$ for small angles. Thus, the propagation function for the ρ component of the field of a vertical dipole, K_{hv}, is equal to $-K_{vv}\sin\psi$. Because of the reciprocity theorem, the function K_{hv} also describes the behavior of the z component of the field of the dipole oriented along ρ. The propagation function for the ρ component of such a dipole, K_{hh}, is equal to $-K_{hv}\cdot\sin\psi = K_{vv}\sin^2\psi$.

4. Propagation Function for a Pulse along a Plane Surface. Using the boundary condition (2.5), the wave propagation (1.1) can be represented in the form

$$\frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} - \frac{1}{c^2\gamma}\frac{\partial^3 F}{\partial t^3} - \frac{1}{c\rho}\frac{\partial F}{\partial t} + \frac{1}{\rho}\frac{\partial F}{\partial \rho} - \frac{F}{\rho^2} = 0 \tag{4.1}$$

From the self-similarity properties of the solution it follows that $F(t, \rho)$ is a function of the single parameter $A = (t - \rho/c)(\gamma c/2\rho)^{1/2}$, and therefore the quantities $\partial F/\partial\rho$ and $\partial F/\partial t$ in (4.1) can be omitted. Transforming next to the self-similar variable, we obtain

$$2\frac{\partial F}{\partial A} + A^2\frac{\partial^2 F}{\partial A^2} + \frac{1}{2}\frac{\partial^3 F}{\partial A^3} = 0 \tag{4.2}$$

Solving (4.2) for the conditions

$$F(0) = 0, \quad (\partial F/\partial A)_{A=0} = 0, \quad \lim_{A\to\infty} F(A) = 2$$

we obtain

$$F(A) = 2(1 - e^{-A^2}) \tag{4.3}$$

We then obtain for the propagation function an expression which agrees with that obtained in [1]:

$$K_{vv} = 2\frac{tc\gamma}{\rho}\exp\frac{-t^2 c\gamma}{2\rho} \tag{4.4}$$

For short times $t \ll t_2$, $K_{vv} \sim tc\gamma\rho^{-1}$. The nature of this characteristic can be understood qualitatively from the following considerations. Because of Huygens principle, the function K_{vv} at the point C (Fig. 1) can be expressed through its value at an intermediate surface, for which it is convenient to choose a plane perpendicular to the interface and to the direction of propagation. Then

$$\frac{K_{AC}(t)}{R_{AC}} = \iint \frac{K_{AB}(t')K_{BC}(t-t')}{R_{AB}R_{BC}}\,dx\,dz$$

Since $ct = 2\sqrt{R^2/4 + \rho^2} - R$, $c\,dt \approx 4\rho\,d\rho/R$. For short times, $K_{AC} \approx z\gamma^{1/2}/Rt^{1/2}$, $z \sim (2cRt)^{1/2}$, so that

$$K_{AC} \approx \frac{\gamma}{\rho}\int_0^t \frac{dx}{\sqrt{x(t-x)}} \sim \frac{tc\gamma}{\rho}$$

One can use the Leontovich conditions to determine the other propagation functions. From (2.1) and (4.4) we obtain

$$K_{hv} = \frac{\partial}{\partial t}\frac{4\alpha A^{3/2}}{\sqrt{\pi}}\int_0^1 e^{-x^2A^2}\frac{x\,dx}{\sqrt{1-x}} \tag{4.5}$$

Since Eq. (2.1) is also valid for K_{hv} and K_{hh}, we then have

$$K_{hh} = \frac{2c}{\rho}(1 - 2A^2)e^{-A^2} \tag{4.6}$$

If K_{hv} is of constant sign and

$$\int_0^\infty K_{vv}(t)\,dt = 2$$

$K_{hv}(t)$ and $K_{hh}(t)$ go to zero for $t \sim t_2$ and

$$\int_0^\infty K_{hv}(t)\,dt = \int_0^\infty K_{hh}(t)\,dt = 0$$

These characteristics result from the fact that the horizontal components of the field lead to the creation of surface currents. Since these currents must not lead to the accumulation of charge in any region of the plane they must be of changing sign and of such a nature that the time integral which gives the field of a static dipole is zero.

5. Propagation Function for a Pulse Signal with Point of Observation at Small Heights. From the fact that the solution is self-similar in the parameters A and B, it follows that the heights can be considered small, i.e., the propagation function can be expanded in series in z up to a height $z \le \alpha\rho$. Such an expansion is of the form

$$K(t, z) = K(t, 0) + z\left(\frac{\partial K}{\partial z}\right)_{z=0} + \frac{z^2}{2}\left(\frac{\partial^2 K}{\partial z^2}\right)_{z=0} + \cdots \tag{5.1}$$

Only quantities defined at the surface z = 0 enter in (5.1), i.e., they can be determined from the value of the function K(t, 0) by means of the Leontovich boundary conditions. For the z component of the field of a vertical dipole, a comparison of the second and third terms shows that the third term is small in comparison with the second; we then obtain for $K_{vv}(t, z)$

$$t_2 K_{vv}(t, z) = 2Ae^{-A^2} + \frac{\sqrt{2}\,\alpha}{\sqrt{\pi A}}\sin\psi\varphi(A)$$
$$\varphi(A) = 1 - 6A^2 I_1(A^2) + 4A^4 I_3(A^2), \qquad I_n(z) = \int_0^1 e^{-x^2 z}\frac{dx}{\sqrt{1-x}} \tag{5.2}$$

For the functions $K_{hv}(t, z)$ and $K_{hh}(t, z)$, we obtain

$$K_{hv}{}^* = \left(\frac{\pi^2\rho^3}{2^5\gamma c^3}\right)^{1/4} K_{hv} = \Phi_0(A) + \alpha\sin\psi\Phi_1(A) + \frac{\alpha}{2}\sin^2\psi\Phi_2(A) \tag{5.3}$$
$$\Phi_0(A) = {}^3/_4 A^{1/2} I_1(A^2) - A^{5/2} I_3(A^2), \qquad \Phi_1(A) = (2A^2 - 3)\,Ae^{-A^2}$$
$$\Phi_2(A) = I_0(A^2) - 29A^2 I_2(A^2) + 38A^4 I_4(A^2) - 8A^6 I_6(A^2)$$
$$\frac{\rho}{c} K_{hh}(t, z) = \Phi_3(A) + 8\sqrt{2}\,\pi^{-1}\alpha\sin\psi\Phi_4(A)$$

$$\Phi_3(A) = (1 - 2A^2)\,e^{-A^2}, \qquad \Phi_4(A) = A^{1/2}(3.25A^2 I_3(A^2) - 1.12 I_1(A^2) - A^4 I_5(A^2)) \tag{5.4}$$

Figure 3 shows the functions $t_2K_{vv}(t, z)$, K^*_{hv}, $\rho/cK_{hh}(t, z)$ for $z_0 = 0$, $z_1 = 0.5$ km, $z_2 = 1$ km and $\alpha = 3 \cdot 10^{-3}$.

A general property of the resultant functions is a fundamental feature at short times which is associated with wave propagation. Since

$$K_{hv} \approx -\sin\psi\ K_{vv},\ K_{hh} \approx \sin^2\psi K_{vv},\ \sin\psi \ll 1$$

the time region in which the wave mode sets in is correspondingly reduced and the term in the expansion in z corresponding to wave propagation contains z to an increasingly higher power. For $K_{hh}(t, z)$, therefore, one can limit oneself to terms linear in z and reject the wave effects.

6. General Formulas for Pulsed Signal Field in Space. In order to find an expression for the field in space, we use the Huygens principle or the Dyson equation [8], which is equivalent to it in this case. In fact the Dyson equation for interaction with a surface has the form

$$K^*(1,2) = K^\circ(1,2) + \int K^*(1,3)\,\Gamma(3)\,K^\circ(1,2)\,d\Gamma(3)$$

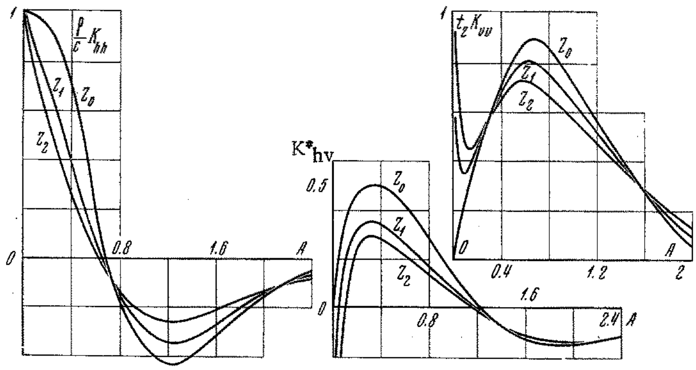

Fig. 3

where K*(1, 2) is the complete function for propagation from 1 into 2, K°(1, 2) is the propagation function without interaction, and Γ(3) is the vortex portion. In the present case, this equation is equivalent to

$$K^*(1,2) = \frac{\delta(t - R_{12}/c)}{R_{12}} + \iint K^*(1,3)\,\Gamma(3)\,\frac{\delta(t' - R_{32}/c)}{R_{32}}\,dS\,dt' \tag{6.1}$$

Integration is carried out over the surface of the ground, point 1 is the source, point 2 the detector, and point 3 the variable of integration. On the other hand, the Green's function which expresses the Huygens-Fresnel principle is of the form [4]

$$U(R,t) = \iint \left(\frac{\partial U(R_{13}, t')}{\partial n}\right)_S \frac{\delta(t' - R_{32}/c)}{R_{32}}\,dS\,dt' \tag{6.2}$$

It is easily seen that Eq. (6.1) agrees with Eq. (6.2) if $\Gamma(3) = \partial(\cdot)/\partial n$. In the case under consideration, the field at the point of observation is

$$E(R,\ t) = 1/\rho K(\rho,\ z,\ t)$$

and the inequalities

$$\sin\psi \ll 1, \quad ct - R_{12} \ll R_{12}$$

are satisfied.

Integrating (6.2) over t', we obtain (see Fig. 4 for notation)

$$E(2) = \int_0^\tau \left(\frac{\partial K}{\partial n}\right)_{z=0} \frac{\rho\,d\theta\,d\varphi}{\theta R_{12}} \tag{6.3}$$
$$\theta = \rho + R_{23} - R_{12}$$

Thus far, the results are valid for any field component and for a dipole of arbitrary orientation. However, integration over φ can be carried out only in specifying the expression for K(t). Taking for K(t) the function $K_{VV}(t)$ from (4.4) and using (2.3), we transform Eq. (6.3) to the form

$$E_z(2) = \frac{-2}{R_{12}} \int_0^\tau \int_0^{\tau - x} \exp\left(-\frac{x^2}{2R_{12}} - \frac{x^2 z^2}{\theta R_{12}}\right) \frac{dx\,d\theta}{\sqrt{\theta(\tau - x - \theta)}} \tag{6.4}$$

It is easy to show that the integral appearing in (6.4)

$$B(y) = \int_0^y e^{-A/\alpha} \frac{d\alpha}{\sqrt{\alpha(y-\alpha)}} = \int_0^y e^{-A/\alpha}\left(\frac{A}{\pi\alpha^3}\right)^{1/2} d\alpha$$

after which Eq. (6.4) can be written as

$$E_z(2) = \frac{-1}{R_{12}} \frac{\partial}{\partial t} \frac{\sqrt{\gamma t}\sin\psi}{\sqrt{\pi}} \int_0^1 \exp\left[-\frac{x^2 t^2 c\gamma}{2\rho} - \frac{x^2 t\gamma \sin^2\psi}{4(1-x)}\right] \frac{dx}{(1-x)^{3/2}} \tag{6.5}$$

This equation agrees with the equation obtained [3] by a Fourier transform of the Sommerfeld solution [5].

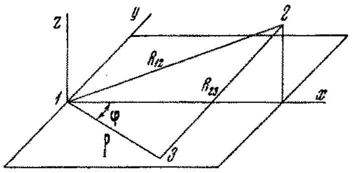

Fig. 4

To obtain equations for the function $K_{hv}(t, z)$ and $K_{hh}(t, z)$, we use Eq. (2.2) and obtain the following relations:

$$K_{hv}(t, z) = \int_0^t \frac{\partial K_{vv}(t', z)}{\partial z} dt' - \sin\psi K_{vv}(t, z) \tag{6.6}$$

$$K_{hh}(t, z) = \int_0^t \int_0^{t'} \frac{\partial^2 K_{vv}(t'', z)}{\partial z^2} - \int_0^t \left(2\sin\psi \frac{\partial K_{vv}}{\partial z} - \frac{1}{\rho} K_{vv}\right) dt' + \sin^2\psi K_{vv}(t, z) \tag{6.7}$$

By simple transformations, it is easy to obtain from (6.6)

$$K_{hv}(t, z) = \frac{\partial}{\partial t} \frac{4}{\sqrt{\pi\gamma t}} \int_0^1 \left(A^2 + \frac{B^2}{1-x}\right) \exp\left[-x^2\left(A^2 + \frac{B^2}{1-x}\right)\right] \frac{x\,dx}{\sqrt{1-x}} \tag{6.8}$$

In the general case, Eq. (6.7) does not reduce to an expression convenient for application. It is easy to show, however, that for small ψ when

$$\frac{\partial^2 K_{vv}}{\partial z^2} \approx \frac{1}{\gamma c^2} \frac{\partial^3 K_{vv}}{\partial t^3} + \frac{1}{c\rho} \frac{\partial K_{vv}}{\partial t}$$

Eq. (5.4) is obtained from (6.7).

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